4085. *Proposed by José Luis Díaz-Barrero. Correction.*

Let ABC be an acute triangle. Prove that

$$\sqrt[4]{\sin(\cos A)\cdot\cos B}+\sqrt[4]{\sin(\cos B)\cdot\cos C}+\sqrt[4]{\sin(\cos C)\cdot\cos A}<\frac{3\sqrt{2}}{2}.$$

We received eight submissions, six of which are correct. We present the solution by Titu Zvonaru.

It is well known that $\cos A+\cos B+\cos C\leq\frac{3}{2}$ [Item 2.16 on p.22 of the book *Geometric Inequalities* by O. Bottema et al; Groningen, 1969]. Using this, together with the facts that $\sin x<x$ for $0<x<\frac{\pi}{2}$, $xy+yz+zx\leq x^2+y^2+z^2$, and $(x+y+z)^2\leq 3(x^2+y^2+z^2)$ we then have

$$\begin{aligned}\sum_{\text{cyc}}\sqrt[4]{\sin(\cos A)\cdot\cos B}&<\sum_{\text{cyc}}\sqrt[4]{\cos A\cdot\cos B}\leq\sum_{\text{cyc}}\sqrt{\cos A}\\&\leq\sqrt{3(\cos A+\cos B+\cos C)}\leq\sqrt{3(\frac{3}{2})}=\frac{3\sqrt{3}}{2}.\end{aligned}$$

Editor's comments. Arkady Alt proved the stronger result that the given upper bound could be replaced by $3\sqrt[4]{\frac{1}{2}\sin\frac{1}{2}}$ which is less than $\frac{3\sqrt{3}}{2}$ since $\sin\frac{1}{2}<\frac{1}{2}$. This new upper bound is attained if and only if the triangle is equilateral. His proof used the Cauchy-Schwarz Inequality, concavity of the functions $\sqrt{\sin x}$ and $\sqrt{\cos x}$, Jensen's Inequality as well as the fact that $\sum\cos A=1+\frac{r}{R}$ and the Euler's Inequality $2r\leq R$.

4086. *Proposed by Daniel Sitaru.*

Let be $f:[0,1]\to\mathbb{R}$; f twice differentiable on $[0,1]$ and $f''(x)<0$ for all $x\in[0,1]$. Prove that

$$25\int_{\frac{1}{5}}^{1}f(x)dx\geq 16\int_{0}^{1}f(x)dx+4f(1).$$

We received seven solutions and present two of them.

Solution 1, by AN-anduud Problem Solving Group.

From the given conditions, f is concave on $[0,1]$. Using Hermite-Hadamard's inequality we get

$$\begin{aligned}16\int_{\frac{1}{5}}^{1}f(x)dx+9\int_{\frac{1}{5}}^{1}f(x)dx&\geq 16\cdot\int_{\frac{1}{5}}^{1}f(x)dx+9\cdot\frac{1-\frac{1}{5}}{2}\cdot\left(f(1)+f\left(\frac{1}{5}\right)\right)\\&=16\int_{\frac{1}{5}}^{1}f(x)dx+\frac{18}{5}f(1)+\frac{18}{5}f\left(\frac{1}{5}\right).\end{aligned}$$

On the other hand, we have

$$f\left(\frac{1}{5}\right)=f\left(\frac{1}{9}\cdot 1+\frac{8}{9}\cdot\frac{1}{10}\right)\geq\frac{1}{9}f(1)+\frac{8}{9}\cdot f\left(\frac{1}{10}\right),$$